Electrodinâmica clássica (2018) - Ficha 1

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1. Análise vectorial

1. Seja $\boldsymbol{\mathscr{r}}$ o vector de separação de um ponto fixo $(x^{'}, y^{'}, z^{'})$ para o ponto (x,y,z), e supondo que $\mathscr{r}$ seja sua magnitude, mostre que :
   (a) $\nabla(\mathscr{r}^2) = 2\boldsymbol{\mathscr{r}}$
   (b) $\nabla(1/\mathscr{r}) = -\boldsymbol{\mathscr{r}}/\mathscr{r}^2$
   (c) Qual é a formula geral para $\nabla(\mathscr{r}^n)$
2. Demonstre que o operador ∇ nas respectivas coordenadas cilindricas e esféricas são dados por :
   (a) $\nabla = \hat{\rho}\frac{\partial}{\partial\rho} + \hat{\theta}\frac{\partial}{\partial\theta} + \hat{k}\frac{\partial}{\partial z}$
   (b) $\nabla = \hat{r}\frac{\partial}{\partial r} + \frac{\hat{\theta}}{r}\frac{\partial}{\partial\theta} + \frac{\hat{\phi}}{r\sin\theta}\frac{\partial}{\partial\phi}$
3. Calcule o gradiente e o laplaciano das funções que se seguem :
   (a) $f(x, y, z) = 2xy + 3\sqrt{y}z^2$
   (b) $f(x, y, z) = y^3 \sin xz + ln2y^3z$
   (c) $f(\rho, \theta, z) = \frac{\rho\cos\theta}{z}$
   (d) $f(\rho, \theta, z) = \frac{\theta z}{\rho^2}$
   (e) $f(r, \theta, \phi) = r\sin 2\theta\cos\phi + r^2 \sin 2\theta\tan\phi$
4. Calcule o divergente e o rotacional das seguintes funções :
   (a) $\vec{f}(x, y, z) = 4xyz\hat{i} - \frac{3z^2}{y}\hat{j} + \cos x\hat{k}$
   (b) $\hat{f}(x, y, z) = y^5 lnxz\hat{i} - \tanh 2y^3z\hat{j} + e^{x^2yz}\hat{k}$
   (c) $\vec{f}(\rho, \theta, z) = \frac{\rho}{z}\hat{\rho} + \sin\theta\hat{\theta} - \rho z\hat{k}$
   (d) $\vec{f}(\rho, \theta, z) = \rho^2\theta z\hat{\rho} + \frac{z}{\tan\theta}\hat{\theta} + 2\rho z\cosh 2\theta\hat{k}$
   (e) $\vec{f}(r, \theta, \phi) = r\sin 2\theta\hat{r} + r\sin^2 2\theta\tan\phi\hat{\theta} - \frac{\phi}{r\cos\theta}\hat{\phi}$
   (f) $\vec{f}(r, \theta, \phi) = \frac{r^3}{ln(\theta\cos\phi)}\hat{r} + 2r\arccos\phi\hat{\theta}$
5. Demonstre as seguintes igualidades :
   (a) $div\ \ grad\Psi = \nabla^2\Psi$
   (b) $rot\ \ rot\vec{a} = grad\ \ div\vec{a} - \nabla^2\vec{a}$
   (c) $(\nabla\vec{a})\ \ \vec{b} = \vec{b}\ \ div\ \ \vec{a} + (\vec{a}\ \ grad)\vec{b}$
   (d) $(\nabla \times \vec{a}) \times \vec{b} = -\vec{b} \times rot\vec{a} - (\vec{a} \times grad) \times \vec{b}$